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Information transfer for characterizing conformation dynamics in network of coupled oscillators

by

Bowen Huang

A thesis submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Major: Electrical Engineering

Program of Study Committee:

Umesh Vaidya, Major Professor

Nicola Elia

Sourabh Bhattacharya

Iowa State University

Ames, Iowa

2016

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DEDICATION

I would like to dedicate this thesis to my mother Xiu Lixin and my father Huang Tiemin, without whose support I would not have been able to complete this work.

I would also like to thank my friends and family for their loving guidance and financial assistance during the writing of this work.

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ACKNOWLEDGEMENTS

I would like to take this opportunity to express my thanks to those who helped me with various aspects of conducting research and the writing of this thesis. First and foremost, Dr. Umesh Vaidya for his guidance, patience and support throughout this research and the writing of this thesis. His insights and words of encouragement have often inspired me and renewed my hopes for completing my graduate education. I would also like to thank my committee members for their efforts and contributions to this work: Dr. Nicola Elia and Dr. Sourabh Bhattacharya.

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ABSTRACT

In this paper, we explore the application of a novel definition of information flow in network dynamical system. The proposed definition of information flow is used for understanding the conformation change in network dynamical systems. Information transfer between network components is used to determine relative contributions of various subsystems in the network to the collective or the emergent dynamics of the network. These relative contributions of information flow from individual subsystems to network collective dynamics allows us to determine which subsystem is most influential for the emergent dynamics of the network. Identification of such influential subsystem can be used to take appropriate local control action at the subsystem level for enhancing or suppressing the network collective dynamics. We provide both model-based and data-driven approaches involving operator theoretic methods for the identification of influential subsystem in the complex network.

The case studies in the IEEE 39-bus power system and I-80 transportation data, presented the potential applications of information transfer in predicting global instability phenomenon and identifying the dominant mode of traffic pattern. The future works focus on the nonlinear system and large scale data for the further application of information transfer.

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CHAPTER 1. GENERAL INTRODUCTION

1.1 Background

Information flow, or information transfer, as it sometimes appears in the literature, refers to the transference of information between two subspaces in a dynamical system through some processes, with one subspace being the source, and another the receiver. Information flow is a fundamental concept in general physics which has applications in a wide variety of disciplines such as neuroscience, material science, atmosphere-ocean science, and turbulence research.

In some previous work, Liang and Kleeman (2007) proposed one rigorous formalism of directional measure of information flow in general dynamic system. The analytical expression of information transfer based on linear system, Sinha and Vaidya (2015), is equipped with the additional information conservation property, which is not satisfied by older formalism. In this computational framework, the directional information transfer between components of a network system reflects the predictability in complex dynamical system, the Granger causality in event-related data, and identify the most influential node in social network. Furthermore, with the knowledge of the probability density function, the formalism could be extended to nonlinear system. In the rest of this paper, we imply this definition by using notion of information transfer.

1.2 Research Motivation

Dynamic behavior among a set of competitors for obtaining a maximum amount of influence from other components in a dynamical network is both important and common phenomena in real world. For example, the application of clustering method in social network is intuitively the identification of most influential node in a network dynamical system. In the view of optimization problem, the goal of clustering is to maximize the amount of information flow among the individual clusters.

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In the other hand, the clustering procedure is highly related to both the predictability of complex system and the causality between state variables in different time steps. Hence, the information transfer could also be applied for the purpose of clustering, prediction, and causality analysis. By observing the information transfer of a properly modeled network system, one can predict and identify the most influential node of the large scale network. Compared with other existing prediction criteria or measures, the advantage of information transfer lies in its flexibility of applications to non-convex problems and the ability to explore the network structure, interesting phenomena like phase transition or conformation change, in a more natural way.

1.3 Thesis Organization

In Chapter 2, this research-based thesis begins with a review of literature relative to older information measure theory, famous applications of corresponding information measure, and classic scenarios in the present study.

Chapter 3 contains the preliminary knowledge of information transfer definition, introducing the net information transfer definition (B.Huang (2016)) and applied the model-based expression to understand the conformation change in the coupled oscillators system. The net information transfer also present the potential in predicting global instability in IEEE 39 bus power system.

Chapter 4 introduces a data-driven method based on Koopman Operator computing information transfer, and applied to the traffic pattern analysis on the I-80 transportation data. The related simulation results are also presented in Chao et al. (2016).

Chapter 5 summarizes all of the conclusions drawn from the thesis and plans for future work in non-linear higher-order dynamical system.

References for each chapters contents are given at the end of the paper.

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CHAPTER 2. REVIEW OF LITERATURE

Since the establishment of the information theory (Shannon (2001)), quantification of information flow has been an enduring problem. The challenge lies in that this is a real physical notion, while the physical foundation is not as clear as those well-known physical laws.

During the past decades, formalisms have been established empirically or half-empirically based on observations in the aforementioned diverse disciplines, among which are mutual information (Vastano and Swinney (1988)), Granger causality (Barnett (2009), Granger (1980)) and transfer entropy (Kaiser and Schreiber (2002), Schreiber (2000)). Particularly, transfer entropy is established with an emphasis of the above transfer asymmetry between the source and receiver, so as to have the causal relation represented; it has been successfully applied in many real problem studies. These formalisms, when carefully analyzed, can be approximately understood as dealing with the change of marginal entropy in the Shannon sense, and how this change may be altered in the presence of information flow (see San Liang and Kleeman (2007), section 4). This motivates our research on the possibility of a rigorous formalism when the dynamics of the system is known.

One thus expects that the concept of information flow/transfer may be built on a rigorous footing when the dynamics are known, as is the case with many real world problems like those in atmosphere-ocean science. And, indeed, X. S. Liang and R. Kleeman (2005) find that, for two-dimensional (2D) systems, there is a concise law on entropy evolution that makes the hypothesis come true. Since then, the formalism has been extended to systems in different forms and of arbitrary dimensionality, and has been applied with success in benchmark dynamical systems and more realistic problems.

Specifically, Sinha and Vaidya (2015) proposed an axiom based definition of dynamical networks, whose expression for the for n-dimensional discrete time linear system is unique, directional, and information conservation property, which is not provided with exsiting definitions of information flow/transfer. The general information transfer definition in this paper will be based on this paper, we investigated the

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conformation change in coupled oscillators system, provide some case studies on the power system network. Subhrajit Sinha (2016) defined information transfer in the control dynamical system. The averaged information transfer from Output to Input in a feedback control system is equal to the Bode integral of the sensitivity transfer function from Output to Input. Hence, Information transfer is connected with the structural controllability and structural observability.

Furthermore, the application of Koopman Operator method Schmid (2010) in computing the information transfer will be introduced in an scenario of transportation data analysis.

In the following chapters, we will give a systematic introduction of the ”Net Information Transfer” and a brief review of some important applications. As a convention in the history of development, the terms information flow and information transfer will be used synonymously. Throughout this paper, by entropy we always mean Shannon or absolute entropy, unless otherwise specified. Whenever a theorem is stated, generally only the simulation result is given and interpreted; for detailed proofs, the reader is referred to the original papers.

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CHAPTER 3. INFORMATION TRANSFER AND CONFORMATION CHANGE IN NETWORK OF COUPLED OSCILLATOR AND POWER SYSTEM

In this chapter, we investigate the problem of conformation change in a network of coupled oscillator system with double well internal potential. Conformation change refers to the phenomena where all the oscillators in the network make a transition from one potential well to another potential well under the influence of external perturbation. We propose a novel approach based on information transfer in network dynamical systems to understand this phenomenon. We consider a heterogeneous network system where the internal dynamics of the oscillators are assumed to be nonidentical and the interconnecting Laplacian can also be asymmetric. The objective is to determine which of the network oscillator is most influential in driving the conformation dynamics. We show that the net information transfer of individual oscillators can be used to determine the most influential oscillator which can drive the entire network from one well to another well of the potential. Three different network topologies and one power system network are used to verify our proposed framework.

3.1 Introduction

The conformation change in coupled oscillators system refers to phenomena where the network of oscillators makes a transition from one potential well to another potential well when perturbed externally through excitation or change in initial condition. This is a fascinating dynamical systems phenomena which have found a variety of applications. In a biological network, it is used to understand conformation change in DNA molecules. In Mezić (2006), dynamical system based approach is proposed to investigate this phenomenon. Most common example of engineering application involving coupled oscillator model is an electric power grid Salam et al. (1984). In network power system conformation change can be used to understand instability and cascade failures in the power system Dobson et al.

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(2007); Y. Susuki (2008) . The focus of this paper is slightly different compared to existing literature in this area. We consider a heterogeneous network of coupled oscillator where the internal dynamics of the oscillators are assumed to be different and the interconnection Laplacian is nonsymmetric. The only common feature of the oscillators is that all the internal dynamics are modeled as a double well potential function. The objective is to determine which of the network oscillators is most influential in driving the conformation change in the network. In particular, we are interested in identifying the leader oscillator which when perturbed can cause the entire network oscillator to follow its state. The problem is similar to the leader selection problem in network dynamical systems. The leader selection problem has been studied in the context of linear network system Patterson and Bamieh (2010); Fardad et al. (2011); Clark and Poovendran (2011); Fitch and Leonard (2013). The optimization-based approach proposed for leader selection in the above reference cannot be applied to our problem because of the nonlinear nature of dynamics involved.

In this paper, we propose a novel approach to analyze the phenomenon of conformation change in a network of coupled oscillators. The approach is based on an information theoretic measure called information transfer Sinha and Vaidya (2015); Vaidya and Sinha (2016). In particular, using analytical expressions of information transfer, we use the information transferred from the different sub-spaces of the state space to identify the most influential oscillator in the network and infer about the conformation change. Roughly speaking, the amount of information transferred from any state (or sub-space, say x) to any other state (or sub-space, say y) gives the amount of uncertainty contribution of x to y. Oscillator l is said to be most influential if the net information transfer (this is defined later) of the oscillator l is the largest. However, calculating the information transfer for non-linear systems requires the knowledge of the probability density function and in general this is not known for non-linear systems. To compute the information transfer in nonlinear network we exploiting the fact that the nonlinearity in the coupled oscillator network is small (of order ϵ) and affects only the internal dynamics of the oscillator. We use the analytical formula for information transfer in linear system to approximate the information transfer in the weakly nonlinear coupled oscillator network. The results obtained using this approximation are verified for the time domain simulations of the nonlinear system. For simulation purposes we use the double well potential (with two minima at ± 1 and a local maxima at 0). We verify the results numerically by initializing the most influential node in the negative well of the potential and all the

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other oscillators in the positive well. We notice that the most influential oscillator is able to pull all the other oscillators out of the positive well and put them in the negative well. The results allow us to understand the role played by the internal dynamics of the oscillators and the network topology in determining the most influential oscillators. We claim that our proposed information based approach can also be applied to identify the influential agent or node in linear network dynamical system. In fact, under the assumption that the dynamical system is linear, we provide an analytical expression for information transfer. This analytical expression of information transfer can be used to understand the influence structure and in the formulation of optimization problems to identify influential agent in linear network system.

The organization of the paper is as follows. In Section 3.2, we provide preliminary and brief overview of the proposed information transfer framework developed in Sinha and Vaidya (2016). Application for the information transfer framework for identifying most influential oscillator with various internal dynamics and network topology is discussed in Section 3.3. Conclusions and discussions are presented in Section 3.4.

3.2 Preliminaries of Information Transfer

In this section, we briefly describe the information transfer framework in discrete dynamical systems of the form

$$z(t+1) = f(z(t)) + \xi(t) \tag{3.1}$$

where $z(t) \in \mathbb{R}^N$, $\xi(t) \in \mathbb{R}^N$ are assumed to be vector valued random variables and $\xi(0), \xi(1), \dots$ are independent random vectors each having the same density g. The mapping $f : \mathbb{R}^N \to \mathbb{R}^N$ is assumed to be at least continuous. Let $z = (z_1, \dots, z_N)^\top \in \mathbb{R}^N$. We are interested in finding the information transfer from state z_i to state z_j, as the system evolves from time step t to time step $t+1$. We denote this transfer by the notation $[T_{z_i \to z_j}]_t^{t+1}$. More generally, we are also interested in deriving the information transfer between the subspaces of the dynamical system. The definition of information transfer is inspired from X. S. Liang and R. Kleeman (2005); A. J. Majda and J. Harlim (2007); Sinha and Vaidya (2015), but, instead of absolute entropy we use the conditional entropy to characterize the

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information transfer. System (3.1) can be written as :

$$\begin{aligned} z(t+1) &= \begin{pmatrix} x(t+1) \\ y(t+1) \end{pmatrix} = \begin{pmatrix} f_x(x,y) \\ f_y(x,y) \end{pmatrix} + \begin{pmatrix} \xi_x(t) \\ \xi_y(t) \end{pmatrix} \\ &:= f(x(t),y(t)) + \xi(t) \end{aligned} \tag{3.2}$$

where $x \in \mathbb{R}^{|x|}$ and $y \in \mathbb{R}^{|y|}$ with $|x|$ denoting the dimension of the x sub-dynamics such that $|x|+|y| = N$. By information, we mean the Shannon definition of information and the information of z at time t is given by the entropy of the probability density $\rho_t(z)$. The expression of information transfer is governed by the propagation of the density function and this propagation is given by the following Perron-Frobenius operator $\mathbb{P} : \mathcal{L}^1 \to \mathcal{L}^1$ Lasota and Mackey (1994)

$$\rho_{t+1}(z) = [\mathbb{P}\rho_t](z) = \int_{\mathbb{R}^N} \rho(\tilde{z})g(z - f(\tilde{z}))d\tilde{z} \tag{3.3}$$

Remark 1 *For notational convenience we will use the notation $'$ to denote the time instant $t+1$, that is, $z(t+1) := z'$ and $z(t) := z$.*

To calculate the information transfer from x to y, we need to study the evolution of the density, not only for (3.2), but also for the modified system, given by

$$\begin{aligned} x' &= x \\ y' &= f_y(x,y) + \xi_y \end{aligned} \tag{3.4}$$

Following X. S. Liang and R. Kleeman (2005), we say that x is frozen (held constant), as the system evolves from time step t to time step $t+1$. Let $\rho_{\not{x}}(y,t)$ be the probability density function with x freezed from time instant t to $t+1$

With this, the information transfer from the x sub-space to the y sub-space, as the system evolved from time step t to time step $(t+1)$ is defined as

Definition 2 *[Information transfer] The information transfer from x to y for dynamical system (3.1) as the system evolves from time t to time $t+1$ (denoted by $[T_{x\to y}]_t^{t+1}$) is given by following formula*

$$\begin{aligned} [T_{x\to y}]_t^{t+1} &= H(\rho(y(t+1)|y(t))) \\ &\quad - H(\rho_{\not{x}}(y(t+1)|y(t)) \end{aligned} \tag{3.5}$$

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where $H(\rho(y)) = -\int_{\mathbb{R}^{|y|}} \rho(y)\log\rho(y)dy$ *is the entropy of probability density function* $\rho(y)$ *and* $H(\rho_{\not x}(y(t+1)|y(t))$ *is the entropy of* $y(t+1)$*, conditioned on* $y(t)$*, where* x *has been freezed.*

Remark 3 *For notational convenience, we will drop the sub-script and super-script and will use the notation* $T_{x\to y}$ *to denote the information transfer from* x *to* y*.*

The above definition of information transfer can be understood by rewriting the expression of information transfer as follows:

$$H(\rho(y(t+1)|y(t))) = [T_{x\to y}]_t^{t+1} + H(\rho_{\not x}(y(t+1)|y(t))) \tag{3.6}$$

According to equation (3.6), the the change in total entropy of y is equal to the transfer from x to y i.e., $T_{x\to y}$ and change in the entropy of y due to itself, that is, when x is freezed. The effect of freezing x is to remove the contribution of x towards the the change in the entropy of y.

Equation (3.5) gives the information transfer from subspace x to subspace y as the dynamical system (3.1) evolves from time step t to time step $t+1$. The general expression of information transfer from state z_i to state z_j is given by

$$[T_{z_i\to z_j}]_t^{t+1} = H(\rho(z_j(t+1)|z_j(t)) - H(\rho_{\not z_i}(z_j(t+1)|z_j(t)) \tag{3.7}$$

A novel outcome of our definition of information transfer is that it can be generalized to define information transfer over n time steps for any $n \in \mathbb{Z}^+$.

3.2.1 n-step Information Transfer

The information transfer defined in Definition (2) gives the information transfer from x to y as the system evolves from time step t to time step $t+1$. So this can be viewed as a *one*-step information transfer. One of the novel advantage of our definition of information transfer is that this definition can be easily extended to define n-step information transfer form x to y, that is, the information transfer from x to y as the system evolves from time step t to time step $t+n$, for $n \in \mathbb{Z}^+$. To define the n-step information transfer, we introduce the following notation

$$y_t^{t+n} = (y(t+n), y(t+n-1), \cdots, y(t)).$$

Then the n-step information transfer is defined as

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Definition 4 (n-step Information Transfer) *The information transfer over $n > 0$ time steps from x to y, denoted as $[T_{x\to y}]_t^{t+n}$, as the dynamical system (3.1) evolves from time step t to time step $t+n$ is defined as*

$$[T_{x\to y}]_t^{t+n} = H\left(\rho(y(t+n)|y_t^{t+n-1})\right) \\ -H_{\not{x}}\left(\rho(y(t+n)|y_t^{t+n-1})\right) \tag{3.8}$$

where $H_{\not{x}}\left(\rho(y(t+n)|y_t^{t+n-1})\right)$ is the conditional entropy of $y(t+n)$, conditioned on its $n-1$ time step past and throughout this evolution, x is held frozen.

With this definition of n-step transfer, one has

Theorem 5

$$\sum_{i=1}^{n}[T_{x\to y}]_t^{t+i} = H(\rho(y_t^{t+n})) - H_{\not{x}}(\rho(y_t^{t+n})) \tag{3.9}$$

The proof is omitted due to space constraints.

3.2.2 Information Transfer in Linear Dynamical System

In linear systems, with additive Gaussian noise, the linearity property allows us to give closed form expressions for the information transfer. Consider the following stochastic perturbed linear dynamical system

$$z(t+1) = Az(t) + \sigma\xi(t) \tag{3.10}$$

where $z(t) \in \mathbb{R}^N$ and $\xi(t)$ is vector valued Gaussian random variable with zero mean and unit variance. We assume that the initial condition is Gaussian with zero mean and covariance $\Sigma(0)$. Since the system is linear, the distribution of the system state for all future time will remain Gaussian with covariance $\Sigma(t)$ satisfying

$$A\Sigma(t-1)A^\top + \sigma^2 I = \Sigma(t)$$

To define the information transfer between various subspaces we again introduce following notation to split the A matrix

$$z(t+1) = \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} A_x & A_{xy} \\ A_{yx} & A_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \sigma\xi \tag{3.11}$$

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The covariance matrix can be similarly split as $\Sigma = \begin{pmatrix} \Sigma_x & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_y \end{pmatrix}$. With this decomposition, it can be shown that the information transfer from x to y is

$$[T_{x \to y}]_t^{t+1} = \frac{1}{2} \log \frac{|A_{yx} \Sigma_y^S(t) A_{yx}^\top + \sigma^2 I_y|}{|\sigma^2 I_y|} \tag{3.12}$$

where $\Sigma_y^S = \Sigma_x(t) - \Sigma_{xy} \Sigma_y(t)^{-1} \Sigma_{xy}^\top$ is the Schur complement of $\Sigma_y(t)$ in $\Sigma(t)$ and I_y is the identity matrix of dimension of y.

The general expression for the information transfer from state z_1 to z_2 can also be found and can be expressed as

$$[T_{z_1 \to z_2}]_t^{t+1} = \frac{1}{2} \log \frac{|A_{z_2 z_{\not 2}} \Sigma_{z_2}^s(t) A_{z_2 z_{\not 2}}^\top + \sigma^2 I|}{|A_{z_2 z_{\not 1 \not 2}} (\Sigma_{z_2}^s)_{z_{\not 1}} A_{z_2 z_{\not 1 \not 2}}^\top + \sigma^2 I|} \tag{3.13}$$

where

$$A = [A_{ij}], i, j = 1, 2, \cdots, N$$

and

$$A_{z_2 z_{\not 2}} := \begin{pmatrix} A_{z_2 z_1} & A_{z_2 z_3} & \ldots & A_{z_2 z_N} \end{pmatrix}$$
$$A_{z_2 z_{\not 1 \not 2}} := \begin{pmatrix} A_{z_2 z_3} & A_{z_2 z_4} & \ldots & A_{z_2 z_N} \end{pmatrix}$$

are row vectors. $\Sigma_{z_2}^s$ and $(\Sigma_{z_2}^s)_{z_{\not 1}}$ are the Schur complement of Σ_{z_2} in Σ and $\Sigma_{z_{\not 1}}$ respectively. The matrix $\Sigma_{\not z_1}$ is the covariance matrix obtained from $\Sigma(t)$ by deleting the row and column corresponding to z_1 state. For a detailed discussion of the definition and basic properties of information transfer see Sinha and Vaidya (2016).

3.3 Information Transfer and Conformation Dynamics

In this section, we present results on the application of information transfer to understand conformation dynamics in network of coupled oscillator systems. In particular, we use information transfer to understand influence structure in network dynamical system and to determine which network component is most influential for causing conformation change in the network of coupled oscillator. The dynamical model for the coupled oscillator system is written as follows:

$$m_k \ddot{\theta}_k = -\epsilon \frac{\partial V}{\partial \theta_k}(\theta_k) - \mathcal{L}_k \theta - d\dot{\theta}_k, \quad k = 1, \ldots, N \tag{3.14}$$

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where m_k and d_k are the mass and damping coefficient of the k^{th} oscillator. V is the internal potential of the oscillators and and assumed to be double well (Fig. 3.1(a)) $\mathcal{L}_k$ is the k^{th} row of the interaction Laplacian and $\epsilon > 0$ is assumed to be small parameter. Let $x_k = (\theta_k, \dot{\theta}_k)$ be the state of the k^{th} agent and x_k^c denotes the state of rest of the agents excluding the k^{th} agents. To determine the influence of k^{th} agent on the rest of the network we will compute the information flow from k^{th} agent to the rest of the network and vice versa, i.e., $T_{x_k \to x_k^c}$ and $T_{x_k^c \to x_k}$. The net information transfer from k^{th} agent to the rest of the network is denoted by NeT_{x_k} and is defined as follows :

$$NeT_{x_k} := T_{x_k \to x_k^c} - T_{x_k^c \to x_k} \tag{3.15}$$

The objective is to use the information transfer from individual oscillator and net information transfer as defined above to determine which oscillator is most responsible for conformation change. Conformation change is defined as a phenomena where one of the oscillator initialized in negative side of the potential well at -1 can pull all the other oscillators of the network, initialized in the positive well at $+1$, to the negative well.

Computing the information transfer for general nonlinear network system will require us to propagate the probability density function. However, propagation of probability density function for general nonlinear system is a challenging problem. We exploit the fact that the nonlinearity in the coupled oscillator system is of order ϵ and it affects only the internal dynamics of the oscillators to approximate the information transfer in the nonlinear network with its linear approximation obtained by taking $\epsilon = 0$, so that the linear approximate model of the network is given by

$$m_k \ddot{\theta}_k = -\mathcal{L}_k \theta - d\dot{\theta}_k, \quad k = 1, \dots, N \tag{3.16}$$

We make use of the analytical expression for information transfer for general linear system as derived in Section 3.2 to determine the information transfer between k^{th} oscillator of the network and its complement and the net information transfer. Although the information transfer is computed using the linear approximation of the nonlinear network, the time domain simulation for verifying the conformation change is obtained using the nonlinear network model.

The double well potential is defined using following parameters

$$V(\theta) = E(C\theta^4 - \theta^2).$$

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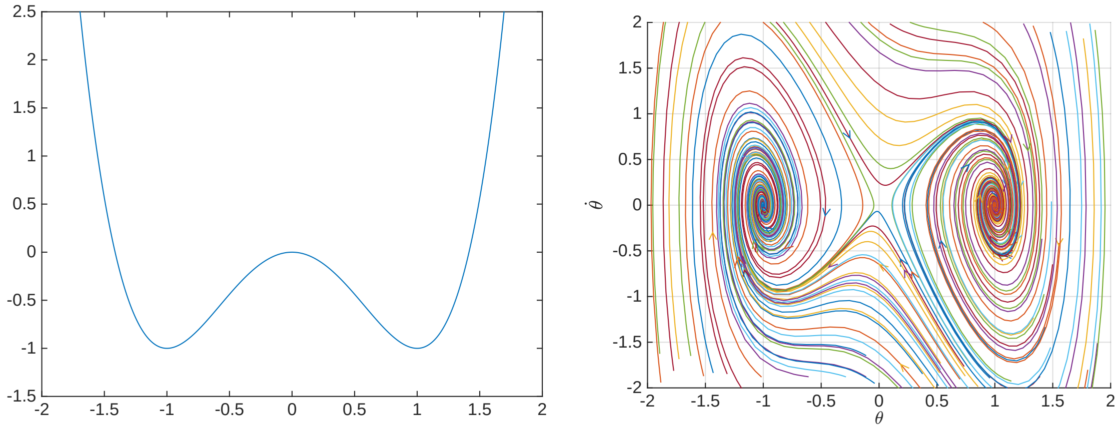

(a) (b)

Figure 3.1 (a) Double well potential. (b) Phase portrait of a single oscillator.

The phase portrait of the single oscillator for the parameter value of $E = 2$ and $C = 0.5$ is shown in Fig. 3.1(b). The phase portrait for a single oscillator system consists of three equilibrium point. The two stable equilibrium points at $(\pm 1, 0)$ correspond to the local minima of the potential function shown in Fig. 3.1(a). The unstable equilibrium point at the origin corresponds to the local maxima of the double well potential. The system will settle at one of the stable equilibrium point depending on the initial conditions. When the oscillators are interconnected through the Laplacian, $\mathcal{L}$, the network system continues to have these three equilibrium points due to the Laplacian property of the interconnection. To which of the two stable equilibrium point the network system will converge to, is now a function of not only the state of the individual oscillator but also the interconnection Laplacian. In particular, the domain of attraction of the stable equilibrium point will be a complicated function of parameters describing internal dynamics of the oscillators and the interconnection Laplacian. In the following we show that the information transferred from individual oscillators to the network and vice versa can be used to understand this conformation change phenomena.

In all the following examples, one agent (say x_1) is initialized at the negative well, while the rest of the oscillators start at the well which has the minima at 1 and we study how agent x_1 influence the rest of the agents to change their conformation, that is, how x_1 makes the rest of the agents cross the potential barrier and flip over to the well with the minima at -1.

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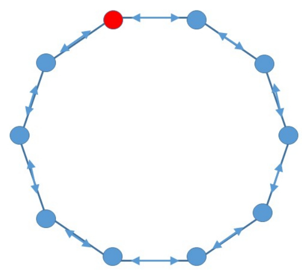
3.3.1 Case I : Nearest neighbour network with nonidentical internal dynamics

The first set of simulations is performed with nearest neighbour topology, as shown in Fig. 3.2. We take 10 oscillators and masses of all the oscillators m_k, $k = 2, \cdots, 10$ are $m_k = .15$ and the mass of oscillator 1 is $m_1 = 3.75$. The damping co-efficient $d = .05$ is assumed to be identical for all the oscillators.



$$\mathcal{L} = \begin{pmatrix} 40 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20 \\ -20 & 40 & -20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -20 & 40 & -20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -20 & 40 & -20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -20 & 40 & -20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & 40 & -20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20 & 40 & -20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -20 & 40 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & 40 & -20 \\ -20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -20 & 40 \end{pmatrix}$$

Figure 3.2 Nearest neighbour topology

We compute the information transfer from each agent to its complement sub-space $(T_{x_k \to x_k^c})$ using the linear system information transfer formula (3.12) and the linear model of the oscillator network (equation (3.16))and this is shown in Table 3.1. We also show the information that each agent receives from its complement $(T_{x_k^c \to x_k})$ and this is shown in Table 3.2. In the tables, the notation o_k is the k^{th} oscillator.

Table 3.1 Information transferred by individual agent

o_1	o_2	o_3	o_4	o_5	o_6	o_7	o_8	o_9	o_{10}
2.1	1.6	1.9	1.8	1.7	1.7	1.7	1.8	1.9	1.6

Table 3.2 Information received by individual agent

o_1	o_2	o_3	o_4	o_5	o_6	o_7	o_8	o_9	o_{10}
.07	2.3	2.2	2.1	2	2	2	2.1	2.2	2.3

While the information transfer and information received values gives some idea about the influential oscillator (the larger the information transfer from an oscillator, more is its influence), these are not sufficient to decide which oscillator is the most influential. However, if we look at the net information transfer for each oscillator (3.15), we find that the most influential oscillator has positive net transfer,

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Table 3.3 Net information transfer of individual agent for $m_1 = 1.25$

o_1	o_2	o_3	o_4	o_5	o_6	o_7	o_8	o_9	o_{10}
2	-0.6	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.6

while all the other oscillators have negative net transfer. The net information transfer for all oscillators are given in Table 3.3. All the values of information transfers given in the tables are for the case when oscillator 1 is able to pull the other oscillators to its own potential well.

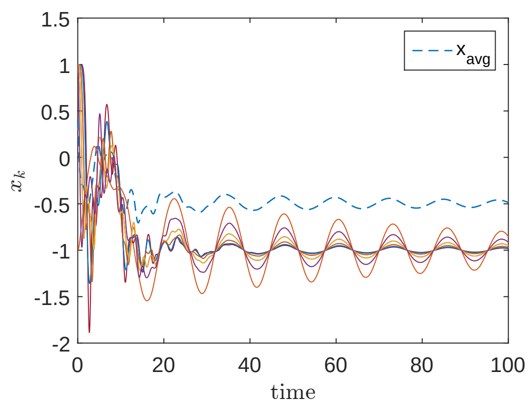

Figure 3.3 The trajectories converge to negative well

Figure 3.3 shows the trajectories of the oscillators when the first oscillator is able to pull all the oscillators to its own well. In this case we chose $m_1 = 3.75$ and the other masses to be $m_k = .15$ and for time domain simulations, we simulated the original system given by equation (3.14) with $\epsilon = .05$ and we find that in this case oscillator 1 is able to pull all the other oscillators to the negative well. In the Figure 3.3 the solid lines correspond to the position of individual oscillator and the dotted line shows the average position of all the oscillators.

However, it is not always that the most influential oscillator can pull all the other oscillators to its own potential well. Depending upon the system parameters, it may happen that the most influential oscillator can pull only a fraction of the oscillators to its own well. Quantifying the minimum amount of information that needs to be exchanged so that the most influential oscillator is able to pull all the other oscillators to its own potential well is an interesting problem and is left for further investigations.

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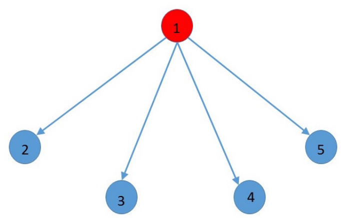
3.3.2 Case II : Leader-follower network with identical internal dynamics

In the second set of simulations, we consider the leader-follower network with identical internal dynamics. In the network, there are directed paths from oscillator 1 to all the other oscillators and there are no other paths. This is shown in Figure 3.4.



$$\mathcal{L} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ -100 & 100 & 0 & 0 & 0 \\ -100 & 0 & 100 & 0 & 0 \\ -100 & 0 & 0 & 100 & 0 \\ -100 & 0 & 0 & 0 & 100 \end{pmatrix}$$

Figure 3.4 Leader-follower network

The masses of all the oscillators are assumed to be 0.5 and the damping co-efficient is $d = 0.05$. In this topology, only oscillator 1 is sending out information and all the other oscillators are receiving information and the net information transfer is positive for oscillator 1 and for all of the other oscillators it is negative (Table 3.4). Hence, we conclude that oscillator 1 is the most influential. The net information transfer, calculated using the approximate linear model (3.16), is shown in Table 3.4.

Table 3.4 Net information transfer

o_1	o_2	o_3	o_4	o_5
3.0557	-2.3659	-2.3659	-2.3659	-2.3659

This conclusion is in fact confirmed by time domain simulations, shown in Figure 3.5, where we find that all the oscillators flip over to the other well and ultimately settle at -1. For the time domain simulations, we use the exact model of the oscillators, given by equation (3.14), with $\epsilon = 0.05$.

Next we modify the topology a little by adding a weak link from oscillator 2 to oscillator 1. This, along with the associated Laplacian is shown in Figure 3.6. Furthermore, the mass of oscillator 2 is increased to 50 and the other masses have masses $m = .5$. The other parameters remain the same. In this case, with the change in the network topology and internal dynamics, oscillator 1 is no longer the most influential, even though it is connected to all the other oscillators. This is verified by the fact that the net information transfer for oscillator 2 is maximum and this means that it is the leader in the

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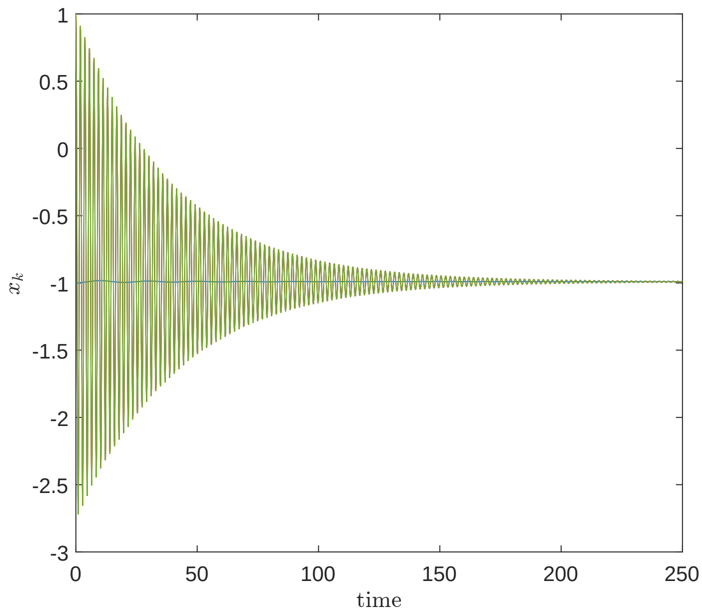

Figure 3.5 Oscillator 1 is the leader

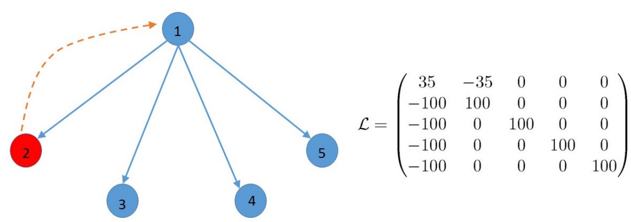

Figure 3.6 Modified leader follower network

network. The net information transfer is shown in Table 3.5.

Table 3.5 Net information transfer

o_1	o_2	o_3	o_4	o_5
1.5727	3.5146	-5.5743	-5.5743	-5.5743

This is again consistent with the time domain simulation of the exact oscillator network model. In this case the second oscillator is initialized at the negative well and all the other oscillators are initialized at the positive well. The trajectories are shown in Figure 3.7. From the figure we find that oscillator 2 does indeed pull all the other oscillators to its own well and hence is the most influential node in the coupled oscillators network.

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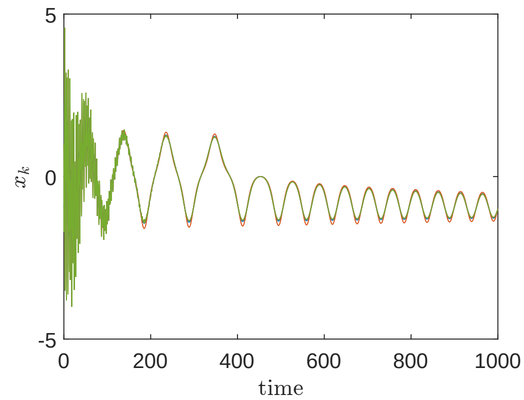
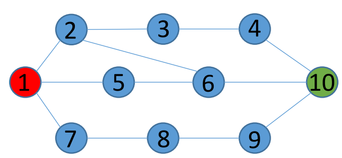


Figure 3.7 Trajectories of the oscillators

3.3.3 Case III : An undirected network

In this set of simulation we consider the undirected network whose topology is given in Figure 3.8. This network has been studied in Zhao et al. (2014).



$$\mathcal{L} = \begin{pmatrix} 30 & -10 & 0 & 0 & -10 & 0 & -10 & 0 & 0 & 0 \\ -10 & 30 & -10 & 0 & 0 & -10 & 0 & 0 & 0 & 0 \\ 0 & -10 & 20 & -10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -10 & 20 & 0 & 0 & 0 & 0 & 0 & -10 \\ -10 & 0 & 0 & 0 & 20 & -10 & 0 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 & -10 & 30 & 0 & 0 & 0 & -10 \\ -10 & 0 & 0 & 0 & 0 & 0 & 20 & -10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10 & 20 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -10 & 20 & -10 \\ 0 & 0 & 0 & -10 & 0 & -10 & 0 & 0 & -10 & 30 \end{pmatrix}$$

Figure 3.8 An undirected network

From the network topology itself, it is not possible to identify the most influential node. For conformation analysis of this network, we assume $m_1 = 10$ and all the other oscillators have mass $m = 1$, the damping co-efficient $d = 0.6$ and $\epsilon = 0.05$. We initialize oscillator 1 in the negative well and all the other oscillators in the positive well. With these initialization, the trajectories of the oscillators are shown in Figure 3.9.

From this we see that agent 1 is able to pull all the other oscillators to the negative well and hence it is the most influential agent. This is again confirmed by the net information transfer for individual agent, where from Table 3.6, we find that only oscillator 1 has a positive net information transfer, whereas, all

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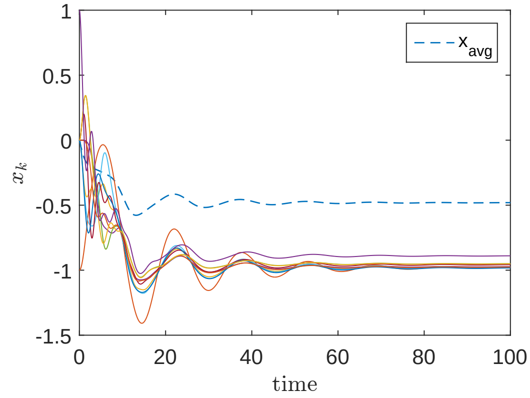

Figure 3.9 Oscillator 1 is most influential

the other oscillators have a negative net information transfer. In this case also, the information transfer is calculated using the approximate linear model of the oscillator network.

Table 3.6 Net information transfer for $m_1 = 10$

o_1	o_2	o_3	o_4	o_5	o_6	o_7	o_8	o_9	o_{10}
0.3	-0.2	-.08	-.08	-0.1	-0.2	-0.1	-.08	-.08	-.2

Table 3.7 Net information transfer for $m_1 = m_{10} = 10$

o_1	o_2	o_3	o_4	o_5	o_6	o_7	o_8	o_9	o_{10}
0.4	-0.3	-.1	-.2	-0.1	-0.3	-0.2	-.1	-.2	.3

In the second set of simulations, we assume that $m_1 = m_{10} = 10$ and the rest of the masses are equal to one. In this case, just by looking at the masses, one can not say which is the most influential oscillator. But from Table 3.7, we find that the net information transfer of oscillator 1 is maximum and so we infer that oscillator 1 is still the most influential oscillator in the network.

This is, in fact, verified by the time domain simulations performed using the exact oscillator model. For this set of simulations, we initialize oscillator 1 at the negative well and all the other oscillators at the positive well. From Figure 3.10, we find that even though $m_{10} = m_1 = 10$, oscillator 1 can pull all the other oscillators over to the negative well and hence it is the most influential. So this example verifies that the net information transfer can be used as a measure to determine the most influential node in a network.

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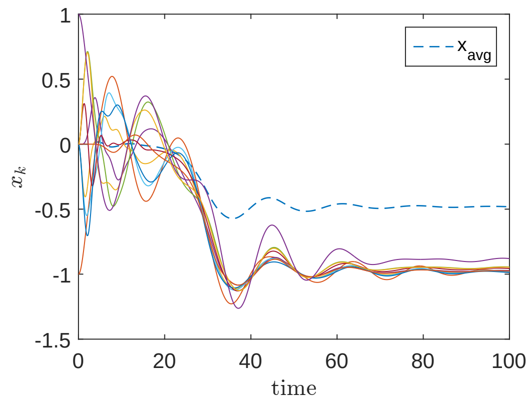

Figure 3.10 Position of the oscillators with $m_1 = m_{10} = 10$.

3.3.4 Case IV : IEEE 39-bus system

In this set of simulation, a more practical application is considered for validation of instability analysis. The IEEE 10-machine 39-bus test system (New England system, Pai (1989)), which is used to implement the proposed scheme, has 10 generators and 46 lines. Its one-line diagram is shown in Figure 3.11(a). As a counterpart of mass of oscillators, the inertia constants of each generators are listed in Table 3.8, where the Generator 2 has largest inertia constant. In the multi-machine power system, due to the topology of the complex network and distribution of inertia, the local disturbance of individual generator may or maynot lead to the global instability, which is undesirable in the real power system.

Table 3.8 Generator inertia constants in the IEEE 39-bus system in seconds

H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}
30.3	500	35.8	28.6	26	34.8	26.4	24.3	34.5	42

To investigate the occurrence of global instability in this power system, we introduce the following system of swing equations with linear coupling,

$$\begin{aligned} \dot{\delta}_i &= \omega_i, \\ m_i\dot{\omega}_i &= p_m - b\sin\delta_i - b_{int}[\mathcal{L}\delta]_i - d\omega_i. \end{aligned} \tag{3.17}$$

The only difference between the system(3.17) and the coupled oscillator system(3.14) is the internal dynamics changing to $\sin(\delta)$, hence to apply the information transfer expression for the linear model, we assume b is a very small value compared with b_int. The parameters for numerical simulations

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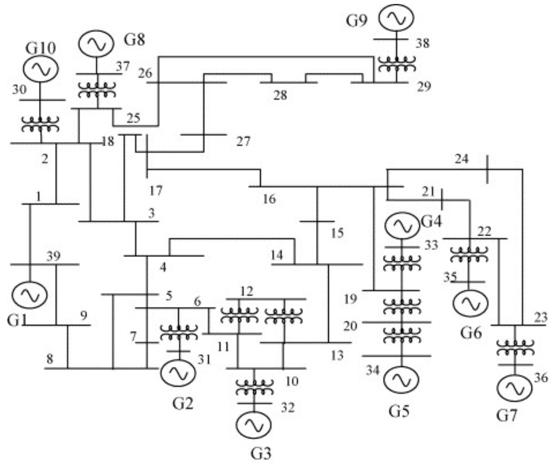

(a)

$$\mathcal{L} = \begin{bmatrix} 32.8 & -2.3 & -2.6 & -1.9 & -0.8 & -2.0 & -1.1 & -11.8 & -3.2 & -7.1 \\ -2.3 & 25.6 & -10.6 & -1.5 & -0.7 & -1.6 & -0.9 & -1.4 & -0.8 & -5.8 \\ -2.6 & -10.6 & 27.0 & -2.1 & -1.0 & -2.3 & -1.2 & -1.6 & -1.0 & -4.6 \\ -1.9 & -1.5 & -2.1 & 33.5 & -16.5 & -4.9 & -2.7 & -1.4 & -1.4 & -1.1 \\ -0.8 & -0.7 & -1.0 & -16.5 & 24.1 & -2.2 & -1.2 & -0.6 & -0.6 & -0.5 \\ -2.0 & -1.6 & -2.3 & -4.9 & -2.2 & 31.0 & -14.0 & -1.4 & -1.5 & -1.2 \\ -1.1 & -0.9 & -1.2 & -2.7 & -1.2 & -14.0 & 23.3 & -0.8 & -0.8 & -0.6 \\ -11.8 & -1.4 & -1.6 & -1.4 & -0.6 & -1.4 & -0.8 & 26.5 & -3.8 & -3.8 \\ -3.2 & -0.8 & -1.0 & -1.4 & -0.6 & -1.5 & -0.8 & -3.8 & 14.4 & -1.2 \\ -7.1 & -5.8 & -4.6 & -1.1 & -0.5 & -1.2 & -0.6 & -3.8 & -1.2 & 25.8 \end{bmatrix}$$

(b)

Figure 3.11 (a) Single-line diagram of IEEE 39-bus New England system. (b) Laplacian matrix.

are in per-unit system and are given as follows: $p_m = 0.009$, $b = 0.01$, $b_{int} = 1$ and $d = 0.05$. The Laplacian matrix of the reduced 10 generators, $\mathcal{L}$ in Figure 3.11(b), is based on the package of *Matpower*(Zimmerman et al. (2011)).

The first set of simulation is implemented under the local disturbance from Generator 1, when $\delta_1(0) = 2.5$, which is outside of the homoclinic orbit. The time domain simulation in Figure 3.12 shows that the Generator 1 is not able to pull the whole system out of the homoclinic orbit Γ_0, even when $\delta_1(0)$ starts from outside of Γ_0, and all the other generators starts from $\delta_i = 1$. From this we see that the individual generator connected to all the others still can not affect most to the global instability. This could be confirmed by the net information transfer for individual generator, where from Table 3.9, we can see that only generator 2 has a positive net information transfer, whereas, all the other generators have a negative net information transfer.

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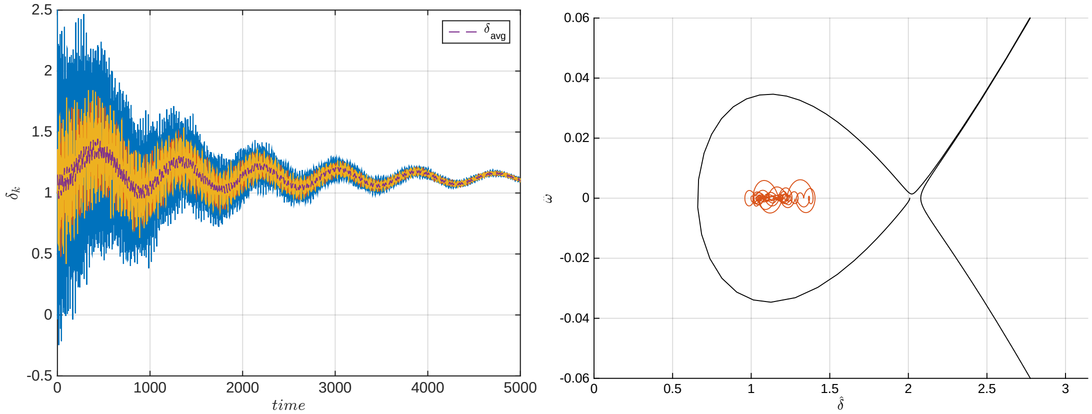


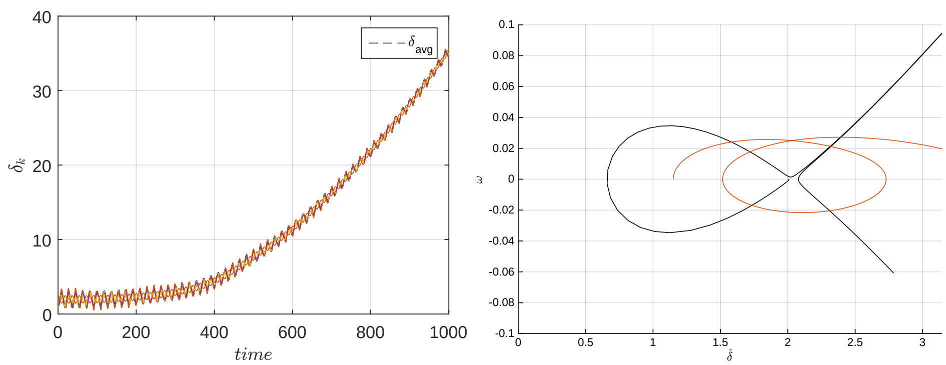
(a) The trajectories of δ_1 to δ_{10} and $\bar{\delta}$ (b) Collective dynamics converges in homoclinic orbit

Figure 3.12 The global stability of system holds when $\delta_1(0) = 2.5$

Table 3.9 Net information transfer for IEEE 39 bus system

g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
-0.7	2.0	-1.3	-0.6	-0.5	-0.4	-0.7	-0.9	-0.7	-0.6

The time domain simulation in Figure 3.13 validates our conclusion from the net information transfer, when we initialize $\delta_2(0)$ from 2.5, which is outside of the homoclinic orbit Γ_0, all the generators are pulled out of the homoclinic orbit. The power system's global instability could be judged by the most influential generator in the complex network, which is reflected by the net information transfer for individual generators. We infer that generator 2 is the most influential generator in the 39 bus system. In this scenario, the distribution of inertia obviously dominates the global instability of the power system.



(a) The trajectories of δ_1 to δ_{10} and $\bar{\delta}$ (b) Collective dynamics shows instability

Figure 3.13 The system goes to unstable when $\delta_2(0) = 2.5$

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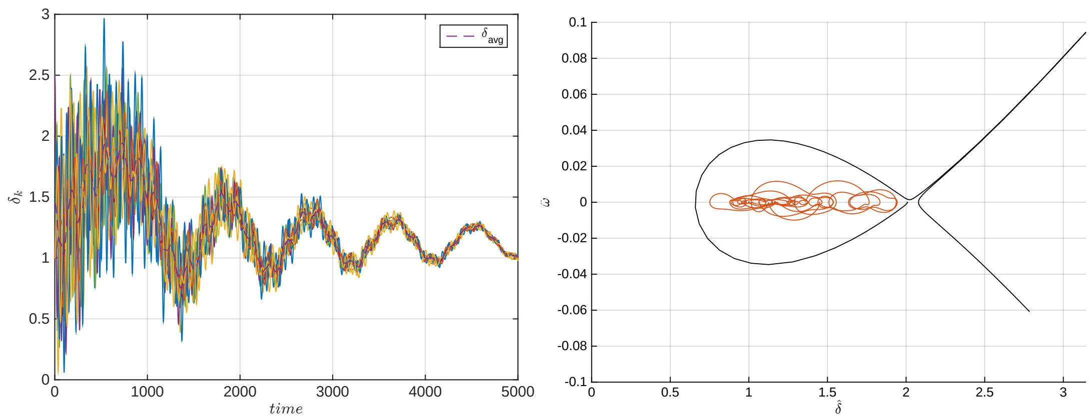
Furthermore, when we increase the number of local disturbance generators, in Figure 3.14, local disturbance from 2 generators (G1, G3) still can't lead to global instability of the power system. In Figure 3.15, 3 generators (G1, G3, G4) under the disturbance can pull the system to be unstable globally. This could also be judged from the net information transfer values. Since $NeT(g_2) + NeT(g_1) + NeT(g_3) = 2.0 - 1.3 - 0.7 = 0$, when 3 generators (G1, G3, G4) are initialized outside of Γ_0, the sum of net information transfer becomes negative. From the net information transfer values, one can conclude that the information values are proportional to the ability of individual generator causing global instability, when the sum of total net information transfer of all the generators goes to negative, the power system tends to be unstable.



(a) The trajectories of δ_1 to δ_{10} and $\bar{\delta}$ (b) Collective dynamics $\bar{\delta}$ vs $\bar{\omega}$

Figure 3.14 The system is still stable when $\delta_1(0) = \delta_3(0) = 2.5$

3.4 Conclusion

We provided novel approach based on information transfer in a dynamical system to identify most influential oscillator responsible for conformation change in a network of weakly nonlinear coupled oscillator system. Identifying the most influential oscillator responsible for conformation change is a complicated function of internal dynamics of the oscillators and the interconnection Laplacian. We show that the net information transfer of individual oscillators captures this complicated function of determining the most influential oscillator. In particular, the oscillator with maximum net information transfer is verified to be most influential and can drive the network state from one potential well to

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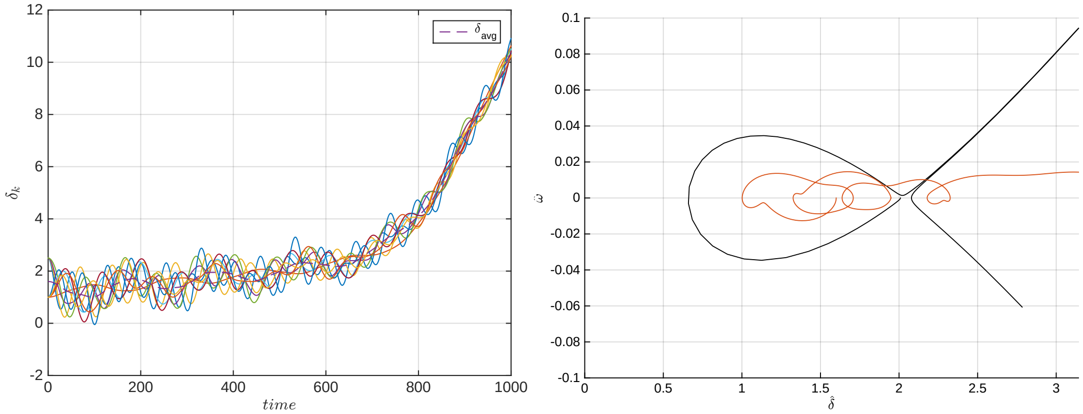

(a) The trajectories of δ_1 to δ_{10} and $\bar{\delta}$ (b) Collective dynamics $\bar{\delta}$ vs $\bar{\omega}$

Figure 3.15 The system becomes unstable when $\delta_1(0) = \delta_3(0) = \delta_4(0) = 2.5$

another. Some interesting results shows the connections between number of unstable generators and global instability in the power system network. Future research work will focus on quantifying the amount of information transfer required for conformation change of the network.

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CHAPTER 4. DATA DRIVEN EXPLORATION OF TRAFFIC NETWORK SYSTEM DYNAMICS USING KOOPMAN OPERATOR-BASED FRAMEWORK

In this chapter, we explore the use of data-driven method to model the system dynamics of an interstate traffic system with high resolution probe data. The dynamic mode decomposition is used to analyze traffic patterns on a 290 mile interstate highway across Iowa. The results show the Koopman Operator-based method method can effectively detect the changes in traffic system dynamics during different time of day. Traffic dynamics during morning peak hours, evening peak hours and off-peak were very different on the studied road. In contrast, the trends over multiple months were similar during the same time periods. The study also found that inclement weather had a significant impact on the system dynamics. In future, the proposed methodology can be used to gain insights in the system dynamics of a traffic network. These models will be instrumental in optimal traffic control, traffic sensor placement and other policy decisions affecting the capacity of the network.

4.1 Introduction

Accurate traffic speed information is important for traffic operation management system. Traffic sensors, like wide range detector and automatic traffic recorder, play a critical role in real-time traffic surveillance and information acquisition. Wide coverage of sensors can provide valuable information for traffic operation, however, the density of traffic sensors is limited to due to the cost of sensors. The implementation of advance traffic control and management system require accurate and reliable real-time traffic condition estimation. Roadside sensors cannot provide enough information if the sensor density is low, especially in rural areas.

The Interstate 80 (I-80) in Iowa is recognized as the Midwest connection between Omaha, Nebraska and Chicago, Illinois. Along this 288.93 centerline mile interstate, the Iowa Department of Transporta-

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tion installed 156 point detectors (as of March 2016). These detectors largely monitor the metro areas and major work zones. The point detectors sensor networks do not scale geographically to monitor the whole interstate system. To monitor the entire I-80 in Iowa, about 867 traffic sensors would be required, assuming that a sensor installed every one third of a mile along this highway, which will be approximately $4.3 million in installation costs.

As an alternative to point sensor, probe vehicle data collected from GPS-enabled vehicles, mobile devices and other sources have been used by many transportation agencies as a supplement of roadside sensor data. Compared to the roadside sensors, probe vehicle data can provide localized information over a larger geographic region as it doesn't require any physical infrastructure Bertini and Tantiyanugulchai (2004).

Agencies have used state wide probe data for performance reporting, sensor spacing and other traffic engineering and management applications Brennan et al. (2013); Feng et al. (2010). Probe data has also been found reliable for congestion detection on freeways Adu-Gyamfi et al. (2015). To deal with the massive scale of the data, most of the past studies either aggregate the probe data at 5-minute or higher averages or focus on smaller geographic region. This paper uses the probe data at its highest resolution across a whole interstate system to train data-driven methods for exploring traffic system dynamics. Data-driven models, dynamic mode decomposition and spatiotemporal feature extraction are used in this paper for understanding the traffic dynamics.

Dynamic Mode Decomposition method (DMD) has been used Schmid (2010) for the dynamical analysis of the fluid flow field data. The basic idea behind this method is to construct a linear system matrix, A, that best explain the time evolving data set $\{z_t\}$ i.e., $z_{t+1} = Az_t$. Spectral analysis of the linear A matrix will help identify spatial temporal coherent structures which are important from dynamical systems perspective. Another data-driven technique, symbolic dynamics Rao et al. (2009), has also been applied in parallel to explore and contrast the understanding of traffic system dynamics.

4.2 Data-driven Analysis Method: Koopman operator-based Framework

In Rowley et al. (2009), linear transfer Koopman operator-based framework is developed for the finite dimensional approximation of the linear operator that best describes the time evolving data set or

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observables. The basic idea behind the framework is a linear, albeit infinite dimensional, representation of nonlinear system. In particular, for nonlinear system $x_{t+1} = F(x_t)$ one can associated linear operator, $\mathbb{U}$, mapping functions or observables $\varphi(x)$ to functions as follows:

$$[\mathbb{U}\varphi](x) = \varphi(F(x)).$$

Note that the data set $\{z_t\}$ can be viewed as a finite dimensional approximation of observables $\varphi(x)$. For the time series data $z_t \in \mathbb{R}^L$ obtained from the transportation network we are assuming that the data is collected from L distributed sensors and there is underlying dynamical system possibly nonlinear that governs the evolution of this data. There are several variant of approximating $\mathbb{U}$, and one of the popular methods is known as Dynamic Mode Decomposition (DMD) Schmid (2010). The basic idea behind the DMD algorithm is to determine a linear mapping A that best connect the time evolving data set i.e., $z_{t+1} \approx Az_t$ for $t = 1, \ldots, N-1$. In the following, we briefly described the DMD algorithm. Let $Z_1^N := \{z_1, \ldots, z_N\}$ be the matrix of snapshots. The constant linear mapping A allows us to write the snapshots as a Krylov sequence $Z_1^N = \{z_1, Az_1, A^2z_1, \ldots, A^{N-1}z_1\}$. For N sufficiently large vectors in Z_1^N become linearly dependent and at this point we can write $z_N = a_1z_1 + \ldots + a_{N-1}z_{N-1} + r = Z_1^{N-1}a + r$, where $a^\top = \{a_1, \ldots, a_{N-1}\}$ and r is the residual vector. We can write the above in matrix form as

$$\begin{aligned} A\{z_1, \ldots, z_{N-1}\} &= \{z_2, \ldots, z_N\} \\ \{z_2, \ldots, z_N\} &= \{z_2, \ldots, Z_1^{N-1}a\} + re_{N-1}^\top, \\ AZ_1^{N-1} &= Z_2^N = Z_1^{N-1}S + re_{N-1}^\top \end{aligned} \tag{4.1}$$

where $a^\top = \{a_1, \ldots, a_{N-1}\}$, $e_{N-1} \in \mathbb{R}^{N-1}$ is unit vector and S is a companion matrix consisting of all zeros except for last column which is equal to a and sub-diagonal entries equal to one. The eigenvalues of S approximate some of the eigenvalues of A. For the robust computation of the spectrum of A instead of computing the matrix S one instead compute matrix $\tilde{S}$ which is related to S via similarity transformation. To achieve robustness one preprocess the time series data using singular value decomposition i.e., $Z_1^{N-1} = U\Sigma W^*$. After substituting the SVD in the Eq. (4.1) and after simplification, we obtain equation and after simplification, we obtain

$$U^*AU = U^*V_2^N W\Sigma^{-1} = \tilde{S}.$$

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The dynamics modes Π_i can then be obtained as follows, $\Phi_i = Uy_i$, where y_i is the i^{th} eigenvector corresponding to the eigenvalue λ_i of matrix $\tilde{S}$ i.e., $\tilde{S}y_i = \lambda_i y_i$. Dynamic mode decomposition method provides a systematic approach for extracting spatial-temporal coherent structures from time series data. Comparing it with proper orthogonal decomposition (POD), which is another popular method for extracting coherent structure, it can be said that while POD attempts decomposition of data set based on orthogonality in space, DMD attempts to represent the data sequence by orthogonalizing it in time.

4.3 Simulation Results

4.3.1 Data

The probe vehicle data used in this study were obtained from INRIX servers INRIX (2016). Real-time probe speeds are available for I-80 in Iowa for more than 90% of the time over the year. Data are reported by road segment. Average vehicle speed on each segment were obtained by 20-second interval. Speed on shorter segments between interchange exit and entrance ramps were removed from the raw dataset to focus on the characteristics of mainline traffic. Data were collected for Jan. and Feb. 2016, and a total of 317 segments on I-80 westbound with an average length of 0.9 mile were studied. For weekdays, the period from 7 AM to 9 AM is defined as morning peak, and the period from 4 PM to 7 PM is defined as evening peak. The time between 10 AM to 3 PM is used as off-peak period. For weekends, data for 5 AM to 10 PM are used in the following analysis.

4.3.2 Dynamic mode decomposition

In order to investigate the dynamic features of system model, we plot the spectrum of the linear, A, matrix obtained from the Dynamic Mode Decomposition analysis. The spectral plot for the month of January during the morning and evening rush time is shown in Fig. 4.1. All the eigenvalues are inside the unit circle indicating stable system dynamics.

In Figs. 4.2 and 4.3 we plot the first two dominant eigenvectors for the Jan. 4 AM and Jan. 25 PM data. Comparing the morning and evening dominant modes we notice that the morning modes has a distinct peak at a fixed spatial location whereas modes obtained from the evening data has no such predominant peak. The spatial location for the dominant peak can be identified right around the

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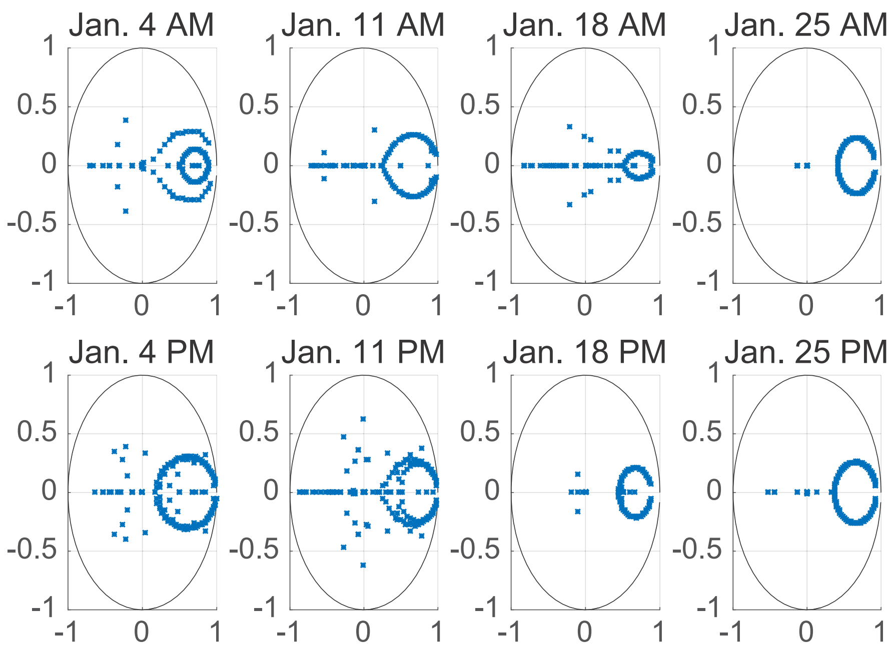

Figure 4.1 Spectrum plots

metropolitan area of Des Moines on the I-80 corridor. Insight gained from the dominant mode can be used for the reduced order modeling and understanding spatial-temporal traffic pattern from the time series data. The difference between the dominant eigenvector for the morning and evening data can be explained using the fact that the morning traffic pattern are more regular compared to evening traffic. Our future research efforts will be to combine the dominant modes and eigenvalues for the purpose of reduced order modeling and for the purpose of optimal sensor placement. In Fig. 4.4, we show the spectrum plot for the day of Feb. 14 AM. Comparing this spectrum plot with Fig. 4.1 and Fig. 4.2, we notice considerable discrepancy. We notice that the eigenvalues plot is quite irregular and there is no one dominant peak in the eigenvector. This discrepancy can be explained by the fact that on Feb. 14 there was snow storm.

Before analyzing traffic patterns in certain periods (e.g. traffic peak hours and off-peak hours, weekday and weekend), it is necessary to investigate the data for detecting special event, such as severe weather condition, local events, and traffic incidents, since these events could significantly change traffic flow characteristics. Under this assumption, similarity among days with similar event will be different from that for any other days. The results are shown in Fig. 4.1, similarity of one day is the sum of

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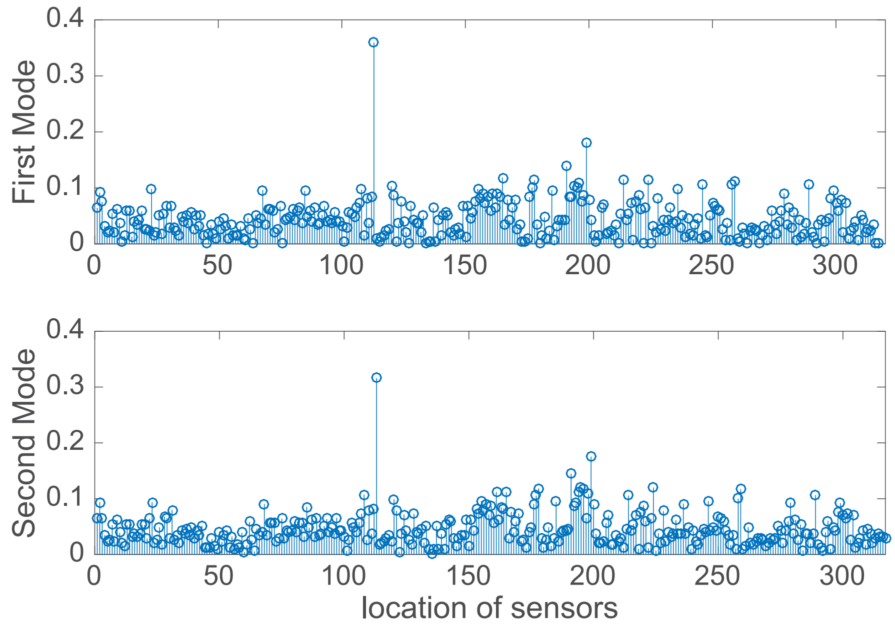

Figure 4.2 First two dominant modes for Jan. 4 AM data

similarities between this day and the rest of days in this month. An obvious decrease of similarity is observed during Jan. 24-26 when a snowstorm produced a average of about 3 inches of snow all over Iowa Arritt (2016). It can be seen that, the similarity metric can be easily applied to detect traffic event. Traffic patterns during peak and off-peak hours are analyzed using the data in Jan. and Feb. 2016, to investigate the traffic flow characteristics during different periods of day.

4.3.3 Discussions

Exploring traffic flow characteristics has great benefits in large-scale traffic control and input-output optimization in sensor placement and other transportation investment planning.

The dynamic mode decomposition, as a robust and reliable algorithm to extract spectrum feature from data sequence, indicates the existence of traffic patterns. Further application of the dominant spatio-temporal structures could extend to pattern recognition for inference and prediction.

Spatiotemproal pattern network, fulfills the task in spatiotemporal feature extraction: (i) *discovering system-wide behavior* for detecting special events, e.g. severe weather condition, and traffic incidents, (ii) *exploring traffic patterns* in different periods of day (e.g., morning peak, evening peak, off-peak hours, weekday, weekend), (iii) *interpreting causality* between segments for finding dominant features

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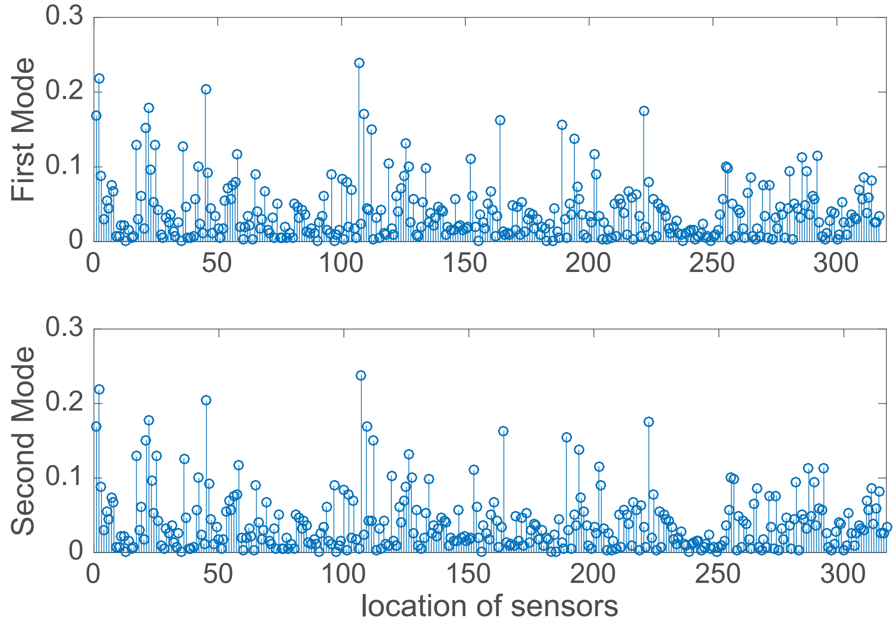

Figure 4.3 First two dominant modes for Jan. 25 PM data

in the dynamic system and quantitatively estimating significance of features with information based metric.

By comparing dominant features in Figs. 4.5 and 4.6, similar phenomenon is observed that fewer dominant features are presented in the morning peak, indicating modeling the morning peak is less complex than the evening peak in this case.

Note, the analysis is limited to the westbound speed data in Jan. and Feb. 2016. More analysis is being implemented with large Archive data for both I80 eastbound and westbound traffic, and integrated with weather and traffic incident data, to detect traffic patterns during different time of day and seasonal trends.

While the current work is focusing on validating the methods for extracting spatiotemporal features in the large-scale system, further works will pursue the following: (i) more analysis and validation in different periods and areas, and (ii) prediction of traffic flow in all segments with optimization strategy in sensor placement.

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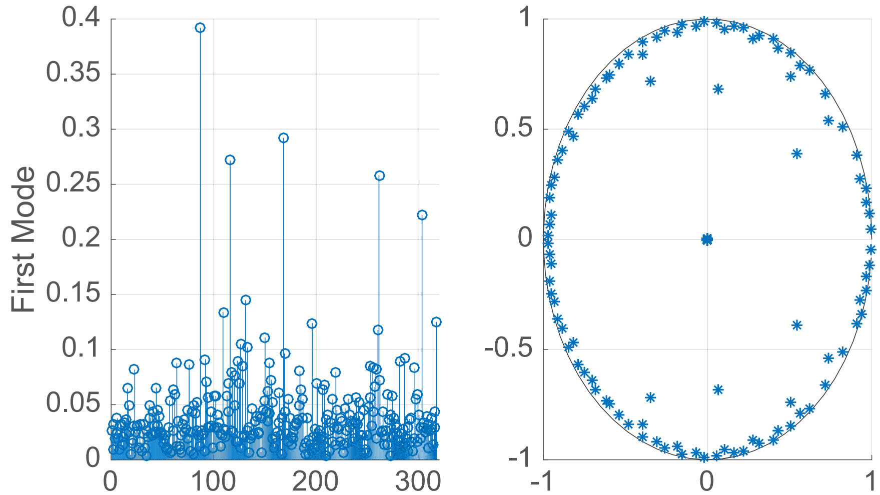

Figure 4.4   Spectrum plot for Feb. 14 AM data

4.4 Conclusions

This paper explored the application of data-driven methods in analyzing transportation system. The data-driven approach discussed here, dynamic mode decomposition, shows significant advances in spatiotemporal feature extraction, especially in: (i) *processing large and high-resolution dataset*, (ii) investigating *dynamic modes* in the system, and (iii) interpreting *spatiotemporal causality* to capture system-wide behavior in diverse situations (e.g., weather condition, traffic patterns).

The analysis of dynamic behaviors in traffic flow could be applied to real-time congestion detection, incident detection, and performance reporting to provide decision support for traffic operation. The characterization of speeds could identify homogeneous road segments and provide reference for optimizing traffic sensor spacing to support transportation agencies in infrastructure investment planning. The deployment of large-scale control strategies for traffic network has increased the need for more accurate and reliable real-time traffic condition prediction. Further modeling based on identified traffic patterns and dominate factors as well as the use of large and high-resolution dataset can improve the accuracy and reliability of traffic flow forecasting, and eventually, enhance the performance of large-scale traffic network management.

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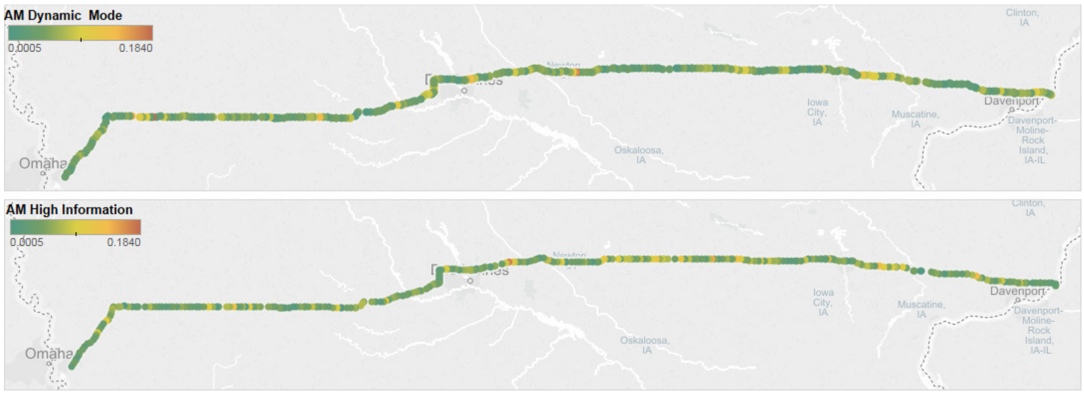
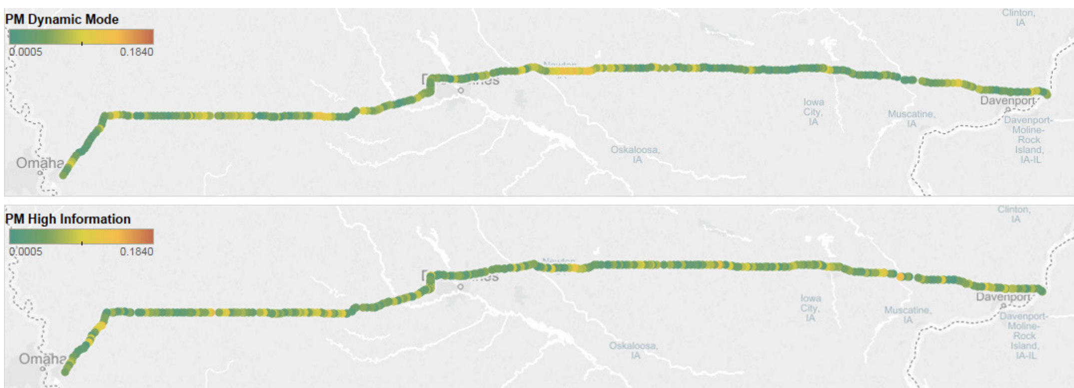


(a) Morning Peak



(b) Evening Peak

Figure 4.5 Dominant eigenvectors for Jan. 4th, 2016

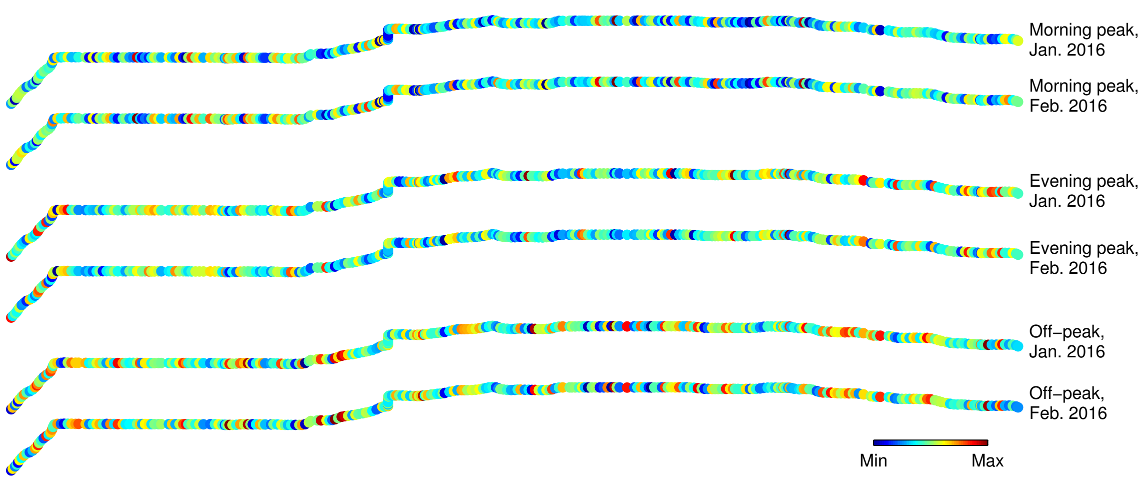


Figure 4.6 Traffic patterns during weekdays along I-80 in Iowa using westbound traffic. The segment in red indicates higher causality, and the segments with the same color represent same causality to the other segments.

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CHAPTER 5. SUMMARY AND DISCUSSION

In the present research, we propose a novel approach to analyze the phenomenon of conformation change in a network of coupled oscillators based on an information theoretic measure called information transfer. We show that the net information transfer of individual oscillators captures this complicated function of determining the most influential oscillator. In particular, the oscillator with maximum net information transfer is verified to be most influential and can drive the network state from one potential well to another.

To investigate further into the phenomenon of conformation change, more case studies in 39 bus power system are implemented. The similar conclusion is drawn from the simulation results. One hypothesis is formalized that net information values be proportional to the ability of individual generator causing global instability, when the sum of total net information transfer of all the generators goes to negative, the power system tends to be unstable.

In the rest of the paper, we explored the application of data-driven methods in analyzing transportation system. The data-driven approach based on Koopman Operator, dynamic mode decomposition, shows significant advances in spatiotemporal feature extraction, especially in: (i) processing large and high-resolution dataset, (ii) investigating dynamic modes in the system, and (iii) interpreting spatiotemporal causality to capture system-wide behavior in diverse situations (e.g., weather condition, traffic patterns).

In the future work, the analysis of dynamic behaviors in multi-agent network system could be applied to real-time detection, clustering identification, and instability prediction in power system. The potential of application in large scale real-time data is also shown in the I-80 transportation scenario. More practical work on the information transfer computation in nonlinear system and higher-order system will be implemented.

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